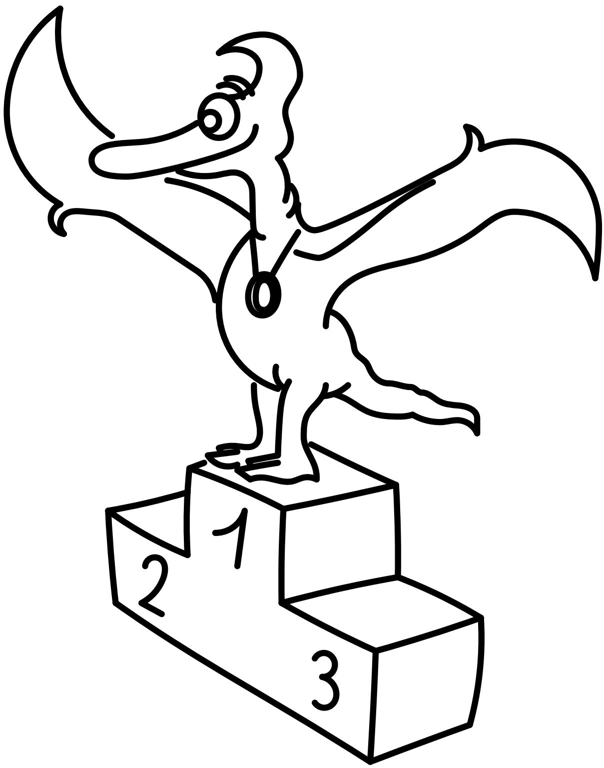
Problems of 11th Physics Brawl



Problem AA ... refreshment 5 – 3 – 2 – 1

We have a cylindrical glass with a base of internal size $S = 9\pi\,\mathrm{cm}^2$. There is some water in the glass with temperature $t_\mathrm{v} = 0\,°\mathrm{C}$ and density $\varrho_\mathrm{v} = 1\,\mathrm{g{\cdot}cm^3}$. Then we put into the water a sphere of ice with radius $r = 2\,\mathrm{cm}$, temperature $t_\mathrm{l} = 0\,°\mathrm{C}$, and density $\varrho_\mathrm{l} = 0.9\,\mathrm{g{\cdot}cm^3}$. After that, the surface of the water in the glass is at height $h = 10\,\mathrm{cm}$. Then we wait until the ice melts. How changes the height of water surface after all the ice is melted and the temperature stays $0\,°\mathrm{C}$? *Lukas Timko wanted participants to think. But that is a serious problem...*

Solution: $\Delta h = 0$ (the surface of the water stays at the same level)

Problem AB ... nonlinear capacitor I 5 – 3 – 2 – 1

Imagine a special nonlinear capacitor. By the nonlinearity, we mean that it is dependent on the voltage applied to the capacitor. What must be the dependence on the capacity to have a constant charge on the capacitor? *Xellos is stealing problems.*

Solution: $C(U) = Q_0/U$ for some constant Q_0

Problem AC ... long-range comms 5 – 3 – 2 – 1

How long does it take for an optical signal to get from the Earth to the Mars? Suppose that both planets orbit the Sun on concentric circular trajectories with radii $r_\mathrm{Z} = 1.50 \cdot 10^8\,\mathrm{km}$ and $r_\mathrm{M} = 2.30 \cdot 10^8\,\mathrm{km}$ and the signal goes straight. (Hint: The answer is an interval.)
Karel went to the cinema on Passengers.

Solution: The solution is interval $\left[\frac{r_\mathrm{M}-r_\mathrm{Z}}{c}; \frac{r_\mathrm{M}+r_\mathrm{Z}}{c}\right] \doteq [270\,\mathrm{s}; 1,270\,\mathrm{s}] \doteq [4.4\,\mathrm{min}; 21.1\,\mathrm{min}]$

Problem AD ... dietetic mistake 5 – 3 – 2 – 1

A glass of orange juice with a volume of $0.2\,\mathrm{l}$ has the energy value of $374\,\mathrm{kJ}$. How much higher is energy value of a glass with the same volume of drink, but with a mixture of juice and vodka in the volume ratio of 1:1? Vodka has $42\,\%$ of alcohol and $1\,\mathrm{g}$ of pure alcohol has the energy value of $29\,\mathrm{kJ}$. For our calculation, we suppose that vodka is a mixture of alcohol and water; water does not have any energy value; the volume of the mixture is equal to the volume of the parts mixed. The density of the alcohol is $\varrho_\mathrm{alcohol} = 790\,\mathrm{kg{\cdot}m^{-3}}$. *Meggy said: "Do not ask..."*

Solution: mixture has more energy of $775\,\mathrm{kJ}$

Problem AE ... I drive two years without an accident 5 – 3 – 2 – 1

Determine the maximum velocity of a motorcycle driving through the sharpest turn of Brno racing circuit in order not to fall or slip. The radius of the turn is $R = 50\,\mathrm{m}$. The coefficient of static friction between tire and asphalt is $f = 0.55$, the motorcycle with the driver has mass of $m = 300\,\mathrm{kg}$ and gravitational acceleration is $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$.
Lukas T. watched Grand Prix.

Solution: $v = \sqrt{fgR} \doteq 16.4\,\mathrm{m{\cdot}s^{-1}} \doteq 59.1\,\mathrm{km{\cdot}h^{-1}}$

Problem AF ... fell down and broke his crown **5 – 3 – 2 – 1**

We drop a body from the height of $h_1 = 100.0\,\mathrm{m}$ and then from the height of $h_2 = 120.0\,\mathrm{m}$. How much higher will be the velocity (we want the velocity difference) in the second case to the first case. The first case is when it is thrown the first time and the body has half of its initial potential energy. The second case is when it is thrown the second time and the body has half of its initial potential energy. Potential energy is zero at the ground level. Neglect air resistance.

Kiki told herself: "Why to count useful things..."

Solution: $\Delta v = \sqrt{g}\left(\sqrt{h_2} - \sqrt{h_1}\right) \doteq 2.99\,\mathrm{m{\cdot}s^{-1}}$

Problem AG ... I drive! **5 – 3 – 2 – 1**

Meggy goes down the hill (inclined plane) on compressed snow. The coefficient of friction between her shoe and snow was $f = 0.25$. As a proper lady, she does not want to disclose her weight. Count the maximum steepness of the hill for Meggy. She wants to stand straight and she does not want to slip. *Meggy slipped down the hill.*

Solution: $\alpha = \arctan f = 0.245\,\mathrm{rad} = 14.04^\circ$

Problem AH ... a ball on a spring **5 – 3 – 2 – 1**

We have a light spring standing perpendicular to the horizontal plane of a table. We have ready a small mass m and we want to put it on the spring. The spring has stiffness k. If we put the mass on the spring from a rest on the top of the spring, what is the maximal shortening of the length of the spring? *Karel looked at a ballpoint pen and a spring.*

Solution: $\Delta l = \frac{2mg}{k}$

Problem BA ... sweetie **5 – 3 – 2 – 1**

Some no-name chocolate has on its wrapping: "milk chocolate 33 %, white chocolate 42 %, dark milk chocolate 25 %". There is also written that cocoa solids content in milk chocolate is at least 30 %, in dark milk chocolate 45 % and white chocolate is without any cocoa solids. The mass of one package is $M = 300\,\mathrm{g}$. If we melt it, how much cocoa solids we have to add to have at least 25 % content of the cocoa solids in the mixture? *Meggy likes sweets.*

Solution: $k = 15.4\,\mathrm{g} = 1.54 \cdot 10^{-2}\,\mathrm{kg}$

Problem BB ... bulletproof **5 – 3 – 2 – 1**

Mikuláš does not like one "potato sack". Once, when they sat (lied down) against each other in wheelchairs, he shot it from a shotgun with pellets of negligible mass with velocity 1 km/s. Who has the higher velocity at the end (after the "potato sack" has been shot) if Mikuláš with shotgun and chair has 80 kg, the "potato sack" has 60 kg with the chair and Mikuláš misses with 1/4 of pellets (but the 3/4 goes right into the "potato sack")? What is the difference between their velocities? The pellets stay in the "potato sack" if they hit it. Neglect air resistance and other resistant forces.

Mikuláš procrastinated from the study of Mathematical Analysis (Calculus).

Solution: They are going to have the same velocity.

Problem BC . . . not to be shot by famous Czech movie... 5 – 3 – 2 – 1

We have an air gun which can shoot a bullet of mass m with velocity v. We are going to shoot to a cylinder of radius R and mass M. The cylinder can freely spin around its main axis (which is vertical), but it cannot move in any other direction. We shoot a bullet from the side and the bullet gets stuck in distance of x from the axis. It is, fortunately, the same distance which would be the minimum distance of the bullet from the axis of the cylinder if the bullet continued its trajectory further. Determine the angular velocity of the cylinder after it has been shot. Neglect air resistance and friction. The cylinder was at rest at the beginning.

Karel was thinking about what to do with an air gun.

Solution: $\omega = \frac{v}{x}\frac{1}{1+\frac{1}{2}\frac{M}{m}\frac{R^2}{x^2}} = \frac{mvx}{mx^2+MR^2}$

Problem BD . . . Lada on the ice 5 – 3 – 2 – 1

Ivan drove his Lada (car) when he found himself accidentally going up the hill with steepness $\alpha = 8^\circ$. His tires are starting to skid. Fortunately, there is no turning point. Determine the minimum velocity of Lada needed to go to the peak of the hill which is $l = 300\,\mathrm{m}$ distant by road. The coefficient of dynamic friction between the tires and the ice is $f = 0.05$.

Michal experienced some snow in Prague.

Solution: $v_0 = \sqrt{2lg(\sin\alpha - f\cos\alpha)} \approx 83\,\mathrm{km{\cdot}h^{-1}}$

Problem BE . . . aircraft 5 – 3 – 2 – 1

At which latitude flies an aircraft when his passengers see the Sun at the same height entire flight? The aircraft flies above the parallel of latitude against the rotation of the Earth with constant velocity $900\,\mathrm{km{\cdot}h^{-1}}$. Suppose, the Earth is a sphere and a day is 24 hours long. Neglect the movement of the Earth around the Sun. *Veronika wanted to stop the time.*

Solution: $\alpha = \arccos\frac{vt}{2\pi r_z} = 57.4^\circ = 57^\circ\,24'$

Problem BF . . . (non)stable reservoir 5 – 3 – 2 – 1

We have a water reservoir. It is being filled by a constant volumetric flow rate of Q_0. The water surface in the reservoir has a stable height of h_0 because there is a hole in the bottom of the reservoir. We will change the volumetric flow rate to $Q = 3Q_0$. After some time the surface is going to stabilize in another height. **How much higher** is the surface going to stabilize (in Q, Q_0 a h_0)? The hole will stay the same size. The reservoir has a constant cross-section and it has sufficient height for both volumetric flow rates.

Karel filled his bathtub... until water started to flow over...

Solution: The surface will be by $\Delta h = \left(\left(\frac{Q}{Q_0}\right)^2 - 1\right) h_0 = 8h_0$ higher.

Problem BG ... a small walk with many buses 5 – 3 – 2 – 1

Daniel decided to walk through Prague along the bus route 177. During his voyage, he met buses going in the other direction once every $t_1 = 6\,\mathrm{min}$ and buses going in his direction in $t_2 = 9\,\mathrm{min}$. Suppose, that the buses and Daniel move with constant speed. What was the **interval of the bus** during his walk? How many times was the bus quicker? *Karel walked around the city.*

Solution: The interval of the bus is $\frac{2t_2t_1}{t_1+t_2} = 7.2\,\mathrm{min}$, the bus moves 5 times quicker than Daniel.

Problem BH ... seven and enough 5 – 3 – 2 – 1

We have a rectangular piece of paper. The paper weight is $\sigma = 100\,\mathrm{g\cdot m^{-2}}$. We also know the density of the paper $\varrho = 600\,\mathrm{kg\cdot m^{-3}}$. How many times we would have to bend the paper in the half to have its thickness increased to higher value than the diameter of the Earth $d = 12\,800\,\mathrm{km}$? *Mirek bent Christmas chocolate wrappings.*

Solution: $n = 37$ bends of paper (precisely, do not take 36 or any other number)

Problem CA ... snow jollifications of Daniel D. 5 – 3 – 2 – 1

Daniel likes to lie in the snow. For simplification, he is naked. It is also often mentioned that one man has thermal output approximately $P = 100\,\mathrm{W}$. What maximal volume of water can Daniel melt in one second? For simplification, the snow has its melting temperature. The heat capacity of water is $c = 4,200\,\mathrm{J\cdot kg^{-1}\cdot K^{-1}}$, the enthalpy of fusion (= latent heat of fusion) is $l = 330\,\mathrm{kJ\cdot kg^{-1}}$. The density of Daniel is $\varrho_{\mathrm{Dan}} = 1,030\,\mathrm{kg\cdot m^{-3}}$, and the density of water is $\varrho = 1,000\,\mathrm{kg\cdot m^{-3}}$. For your calculation, let's hope, that his temperature will not decrease and stays the same. *Karel almost froze on the way to the bus.*

Solution: $\frac{V}{t} = \frac{P}{\varrho l} = 3.0 \cdot 10^{-7}\,\mathrm{m^3\cdot s^{-1}} = 0.30\,\mathrm{cm^{-3}\cdot s^{-1}}$

Problem CB ... thermodynamic exercise 5 – 3 – 2 – 1

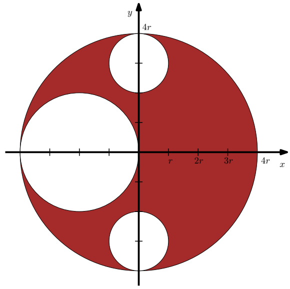
The ideal gas went through an adiabatic process from the state A to B and after that by isothermal process from state B to C. In the first process, the work done was $W_1 = 4\,\mathrm{J}$. In the second process, the work done was $W_2 = 5\,\mathrm{J}$. Determine the change of internal energy of the gas between A and C. *Xellos and a test from Physics I at our Faculty.*

Solution: $\Delta U = -4\,\mathrm{J}$ (the sign minus is important)

Problem CC ... artistic table 5 – 3 – 2 – 1

Where is the centre of mass of the table desk you can see on the picture? The desk can be approximated as homogeneous with (area) density σ. The radius of the smallest circle is r and radius of the desk is $4r$.

Karel thought about lucrativeness of modern arts.



Solution: $[x_{\mathrm{T}}; y_{\mathrm{T}}] = [0.8\,r; 0]$

Problem CD ... electron closure 5 – 3 – 2 – 1

How many times should increase the gravitational constant G (from Newton's law of universal gravitation) to decrease the radius of electron "trajectory" in hydrogen atom ^{1}H by 10 % in comparison to the present value? Use Bohr model of atoms. Suppose that constant k in Coulomb's law stays the same. *Karel was thinking how to combine astrophysics and particle physics.*

Solution: It must be approximately $2.52 \cdot 10^{38}$times higher

Problem CE ... dragon's throat 5 – 3 – 2 – 1

Green dragon Flagon consumes charcoal to have enough fuel for his flame breath. The burning goes according to equation $C + O_2 \longrightarrow CO_2$ which has energy balance $\Delta H = -393.51\,\mathrm{kJ{\cdot}mol^{-1}}$. Determine the temperature of his breath in Celsius. The charcoal is burned adiabatically in air containing 80 % nitrogen and 20 % oxygen and the amount of carbon is equimolar to the amount of molecular oxygen. You also know heat capacities $c_{\mathrm{N}_2} = 29.125\,\mathrm{J{\cdot}mol^{-1}{\cdot}K^{-1}}$, $c_{\mathrm{O}_2} = 29.355\,\mathrm{J{\cdot}mol^{-1}{\cdot}K^{-1}}$ and $c_{\mathrm{CO}_2} = 37.110\,\mathrm{J{\cdot}mol^{-1}{\cdot}K^{-1}}$. Suppose that the heat capacities are temperature independent. The dragon is a reptile, therefore the temperature of his body is equal to the temperature of the environment $t_1 = 25\,^\circ\mathrm{C}$. We want the answer in °C!

Our Catherine was fascinated by dragons. Dracarys!

Solution: $t_2 = \frac{-\Delta H + \left(4c_{\mathrm{N}_2} + c_{\mathrm{CO}_2}\right)t_1}{4c_{\mathrm{N}_2} + c_{\mathrm{CO}_2}} \doteq 2,590\,^\circ\mathrm{C}$

Problem CF ... how to become enlightened 5 – 3 – 2 – 1

We have two samples of radioactive material. Just call them A(lice) and B(en). Sample A has three times longer half-life than B. In the sample A, there is four times higher mass of radioactive element than in B. The ratio of relative atomic masses of radioactive elements in the samples of A to B is 3 to 2. What must be the ratio of distances of the detector from the samples $r_{\mathrm{A}}/r_{\mathrm{B}}$ to register the same number of particles from both samples? Imagine ideal all-directional detector. Suppose the samples are points and isotropic. We are interested about some short time interval; therefore, the activity of the samples is constant. The decay of both samples is of the same type. *Karel thought of a radiology.*

Solution: $\frac{r_{\mathrm{A}}}{r_{\mathrm{B}}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} \doteq 0.943$

Problem CG ... king's crown 5 – 3 – 2 – 1

A king wants a new crown, so he orders someone to make it. Unfortunately, he chose some scammer who wants to make it from some gilded metal. How long time does it take to the scammer to gild the king's crown if he uses golden ions 3+, electrical current $I = 5\,\mathrm{A}$ and wants to have $d = 1\,\mathrm{mm}$ thick layer of gold? The crown has a surface area of $S = 12\,\mathrm{dm}^2$. Suppose, that the ions are deposited evenly on the crown. Faraday constant is $F = 9.65\cdot 10^4\,\mathrm{C\cdot mol^{-1}}$. The heat capacity of gold is $25.4\,\mathrm{J\cdot mol^{-1}\cdot K^{-1}}$. The molar volume of gold is $V_\mathrm{m} = 1.02\cdot 10^{-5}\,\mathrm{m^3\cdot mol^{-1}}$. We want the answer in days. *Our Cate sometimes thinks about very nasty things...*

Solution: $t = \frac{FSdz}{V_\mathrm{m}I} \doteq 7.9\,\mathrm{days}$

Problem CH ... coating 5 – 3 – 2 – 1

We have evacuated vacuum container ($p = 10^{-3}\,\mathrm{Pa}$). We want to use it for a creation of thin layer, $h = 10\,\mathrm{nm}$ thick, of silver on our sample. The sample is $l = 125\,\mathrm{mm}$ distant from the place where is silver evaporated. How long silver wire we need for it? Suppose that silver is deposited evenly in the half-space – upper the place where is the silver evaporated.

Miso "played" with a vacuum apparatus.

Solution: $5\,\mathrm{mm}$

Problem DA ... sand, sand, more sand 5 – 3 – 2 – 1

We are in a desert and we are digging a hole. From which depth it would be energetically cheaper not to bring it up but to evaporate it? The heat capacity of the sand is $0.74\,\mathrm{kJ\cdot kg^{-1}\cdot K^{-1}}$, its enthalpy of fusion (= latent heat of fusion) is $128\,\mathrm{kJ\cdot kg^{-1}}$ and its enthalpy of vaporization (= latent heat of evaporation) is $4,715\,\mathrm{kJ\cdot kg^{-1}}$. Sand boils at temperature $3,223\,\mathrm{K}$. Suppose constant temperature in the hole of $50\,^\circ\mathrm{C}$ and that the gravitational acceleration does not depend on the depth. Suppose that the evaporated sand rises spontaneously from the hole.

Mikuláš likes to cook sand.

Solution: The depth is approximately $712\,\mathrm{km}$.

Problem DB ... Planck frequency 5 – 3 – 2 – 1

Determine the Planck frequency. This is the frequency you can obtain by dimensional analysis (some combination of the right power of some constants) of three fundamental constants: Planck constant $\hbar = 1.05\cdot 10^{-34}\,\mathrm{kg\cdot m^2\cdot s^{-1}}$, gravitational constant $G = 6.67\cdot 10^{-11}\,\mathrm{kg^{-1}\cdot m^3\cdot s^{-2}}$, and speed of light $c = 3.00\cdot 10^8\,\mathrm{m\cdot s^{-1}}$. A multiplicative constant cannot be determined by dimensional analysis. Suppose that the constant is 1. *Karel likes Planck units.*

Solution: $f_\mathrm{P} = \sqrt{\frac{c^5}{\hbar G}} \doteq 1.86\cdot 10^{43}\,\mathrm{Hz}$

Problem DC ... countable 5 – 3 – 2 – 1

How many sand particles of diameter $d = 1\,\mathrm{mm}$ would have we need in order to make $m = 6\cdot 10^{24}\,\mathrm{kg}$ of glass with density of $\varrho = 2.5\,\mathrm{g\cdot cm^{-3}}$. State the *count of digits* of the result.

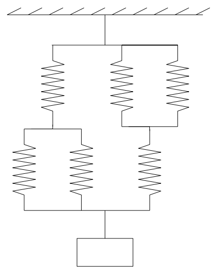
Jan P. loves logarithms.

Solution: The digits count is 31, sand particles are $4.6\cdot 10^{30}$.

Problem DD ... stiff and even stiffer **5 – 3 – 2 – 1**

What is the total spring constant of the system you can see on the picture? Each individual spring has spring constant k.

Karel spoke about a deceased man.



Solution: $k_{\text{total}} = \frac{4}{3}k$

Problem DE ... to Litfago them well **5 – 3 – 2 – 1**

A rope is put over a fixed pulley. The radius of the pulley is $R = 30\,\mathrm{cm}$. The mass of the rope is $m = 300\,\mathrm{g}$ and its length is $L = 4\,\mathrm{m}$. One of the ends of the rope is $L/2$ higher than the second. There are also weights attached to both ends of the rope. The lower end has weight with a mass $m_1 = 250\,\mathrm{g}$ and the higher end $m_2 = 50\,\mathrm{g}$. Calculate the acceleration of the system at the described moment as multiple of the gravitational acceleration g. Neglect the friction and the buoyancy. *Xellos is irritated by walking up the stairs.*

Solution: $a = \dfrac{7}{12}g \doteq 0.58g$

Problem DF ... bolometer says, that... **5 – 3 – 2 – 1**

The solar constant K was determined using bolometers placed on satellites near to the Earth. The solar constant is solar electromagnetic radiation per unit area that would be incident on a plane perpendicular to the rays at a distance of 1 AU from the Sun. Determine the surface temperature of Sun T, knowing Sun's radius R_{S} and mean distance between Sun's and Earth's centres r. *Karel looked at the Sun.*

Solution: $T = \sqrt[4]{\frac{K}{\sigma}}\sqrt{\frac{r}{R_{\mathrm{S}}}}$

Problem DG ... is there a free place? **5 – 3 – 2 – 1**

There is a line of double-chairs in the bus. People sit down every time to a place where is free entire double-chair and only after that all double-chairs have someone sitting on them, people start to sit on the second places of double-chairs. There are g single places (even number to have only double-chairs) and people are $N \leq g$. How many possibilities are there to occupy a bus for N people? We do not distinguish left and right side of double-seats and the people are indistinguishable. We want the solution only for $g_1 = 40$ and $N = 17$, and $g = 40$ and $N = 22$.

Mirek regularly goes by quantum buses.

Solution: $1{,}140$ possibilities for $N = 17$ and 190 possibilities for $N = 22$.

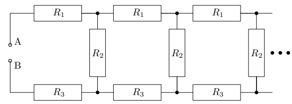
Problem DH ... an accident in LEP for the first time 5 – 3 – 2 – 1

Once upon a time, there was an accelerator called LEP in CERN. What was power delivered by the beam of electrons in this accelerator when the entire beam was focused to some material? The average electric current of the beam was $I = 2\,\mathrm{mA}$, energy of electrons $E = 50\,\mathrm{GeV}$, and circumference of accelerator was $o = 27\,\mathrm{km}$. The answer must be in watts.

Karel remembered long hours spent with Physics V.

Solution: $P = \frac{IE_{\mathrm{e}}}{e} = 100\,\mathrm{MW} = 1.0 \cdot 10^{8}\,\mathrm{W}$

Problem EA ... never-ending triviality 5 – 3 – 2 – 1



Determine the total electric current going through the circuit that you can see on the picture when we put voltage $U_{\mathrm{AB}} = 4.50\,\mathrm{V}$ between points A and B. The resistance of resistors are $R_1 = 1.00\,\Omega$, $R_2 = 2.00\,\Omega$, and $R_3 = 3.00\,\Omega$. The circuit is infinite - the resistors R_1, R_2 and R_3 are repeating for infinite times.

Karel wanted some infinite circuit.

Solution: $I = \frac{U}{\frac{R_1+R_3}{2}+\sqrt{\left(\frac{R_1+R_3}{2}\right)^2+(R_1+R_3)R_2}} \doteq 0.82\,\mathrm{A}$

Problem EB ... the last story of Mr Ballie 5 – 3 – 2 – 1

Let's have some Mr Ballie. He is a homogeneous full sphere with radius $r = 0.50\,\mathrm{m}$. He can be quite well approximated as a black body. The temperature of the environment surrounding him is $t_{\mathrm{p}} = 5.0\,^{\circ}\mathrm{C}$. The temperature of Mr Ballie is, for now, $t_{\mathrm{K}} = 36.0\,^{\circ}\mathrm{C}$. But he could get cold very soon! Therefore, he came up with an (idiotic) idea. He heated himself to the temperature of the highest fever $t_{\mathrm{h}} = 42\,^{\circ}\mathrm{C}$. What is the ratio of the new thermal flux from Mr Ballie P_{h} to the previous P_{K}? We want $\kappa = P_{\mathrm{h}}/P_{\mathrm{K}}$.

Karel wanted a heating in the woods.

Solution: $\kappa = \frac{T_{\mathrm{h}}^4}{T_{\mathrm{K}}^4} \doteq 1.080$

Problem EC ... a proton fell down 5 – 3 – 2 – 1

What kinetic energy in eV gains the proton by falling from infinite distance to the surface of the Earth? The radius of the Earth is $R_{\mathrm{Z}} = 6{,}378\,\mathrm{km}$, Earth mass is $M_{\mathrm{Z}} = 5.98 \cdot 10^{24}\,\mathrm{kg}$ and invariant energy of a proton is $m_{\mathrm{p}}c^2 = 938.3\,\mathrm{MeV}$.

Karel looked on the tests from particle Physics.

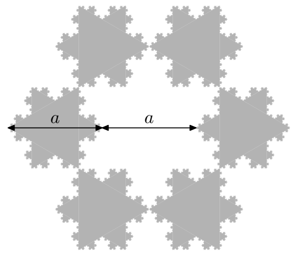
Solution: $E_{\mathrm{k}} = G\frac{m_{\mathrm{p}}M_{\mathrm{Z}}}{R_{\mathrm{Z}}} \doteq 0.65\,\mathrm{eV}$

Problem ED ... overheated 5 – 3 – 2 – 1

Faleš has a computer with a processor of maximal heat power 90 W. There is a copper plate between the processor and cooling system. The thickness of the plate is 3 mm and its cross-section is $400\,\mathrm{mm}^2$. Calculate temperature difference between the top and the bottom of the plate. Heat resistivity of copper is $\varrho = 2.6\,\mathrm{mm{\cdot}K{\cdot}W^{-1}}$. *Faleš is always hot.*

Solution: $\Delta T = 1.755\,\mathrm{K}$

Problem EE ... paved **5 – 3 – 2 – 1**



We wanted to make a new pavement using Koch snowflakes as paving stones. We have two sizes a and $a\sqrt{3}$ of Koch snowflakes, see on the picture. After finishing the pavement, the workers started to throw one left smaller Koch snowflake at each other. What is the moment of inertia of this snowflake? You know that it has mass m.
Lukáš was bothered by the fact that Sierpinski triangle is massless.

Solution: $J = \frac{1}{11} m a^2$

Problem EF ... conflict of data formats **5 – 3 – 2 – 1**

You surely know that computers use some representation of the date and then show it to us in some nicer format (for people). For example, Excel supposes that the day 1 is 1. 1. 1900, day 2 is 2. 1. 1900 and so on. More precisely, 1.00 is 1. 1. 1900 0:00. What is the day according to Excel right now? We want the answer to one decimal place.
"Hint": Excel supposes that the year 1900 was a leap year (wrongfully) and that 2000 was also leap year (rightfully). *Karel was inspired by Excel data format.*

Solution: The solution depends on the time when the participant handed it in. The following is for 17th Feb 2017 when it was held Physics Brawl. From the start to 10:44 was the right answer $42,783.4$. From 10:45 to 10:50 organizers tolerated both solutions $42,783.4$ and $42,783.5$. From 10:51 to 13:09 worked $42,783.5$. From 13:10 to 13:15 one of $42,783.5$ or $42,783.6$. From 13:16 until the end of the competition was the only right solution $42,783.6$.

Problem EG ... Millikan experiment **5 – 3 – 2 – 1**

The precise value of the elementary charge could be determined by Millikan experiment. In this experiment, we measure velocities of charged drops of oil (with mass m and density ϱ_k). These drops are moving between two electrodes. These electrodes create homogeneous electric field E which is alternating in the two directions - up and down. The drops are moving in the direction of the electric field. You know the velocities of droplets v_1, v_2 for both directions of the electric field. Calculate the electric charge q on one droplet. The air density is ϱ_{air}, the buoyancy is not negligible and you can count the air drag using $F_{\mathrm{t}} = 6\pi\eta r v$, where η is the dynamic viscosity of air and r is the radius of a droplet, which you do not know.

Veronika loved that experiment.

Solution: $|q| = 3\pi\eta \frac{v_1+v_2}{E} \sqrt{\frac{9\eta(v_1-v_2)}{4g(\varrho-\varrho_{\mathrm{vz}})}}$

Problem EH ... an accident in LEP for the second time 5 – 3 – 2 – 1

What was the total energy of electron beam of LEP (Large Electron–Positron Collider) in CERN? The average electric current of the beam was $I = 2\,\mathrm{mA}$, energy of each electron $E = 50\,\mathrm{GeV}$, and circumference of the accelerator was $o = 27\,\mathrm{km}$. We demand the answer in kilocalories $1\,\mathrm{kcal} = 1\,\mathrm{Cal} = 4,200\,\mathrm{J}$.

Karel remembered Physics V of Faculty of Mathematics and Physics.

Solution: $E_{\mathrm{s}} = \frac{I\ E_{\mathrm{e}} o}{ec} \doteq 2.1\,\mathrm{kcal}$

Problem FA ... nonlinear capacitor II 5 – 3 – 2 – 1

We have a circuit with a capacitor. This capacitor has capacity dependent on the applied voltage $C(U) = \frac{k}{U_0 - U}$. The capacitor is connected in series with a resistor with resistance R, a switch and a DC source. The voltage of the source of the current is U_0. Calculate needed time t after switching on the switch when the voltage on the capacitor is $U_0/2$.

Xellos forged a problem.

Solution: $\frac{6kR}{U_0}$

Problem FB ... flexible water from a space station 5 – 3 – 2 – 1

There is a sphere of water in the space station in the state of weightlessness. It is disturbed by a touch. The touch starts the movement of the centre of mass of the sphere with speed $2\,\mathrm{cm{\cdot}s^{-1}}$ and the drop starts to oscillate between two ellipsoid shapes. The maximum prolongation of the sphere is $10\,\%$ during the oscillation period.

What is the total kinetic energy obtained by the sphere by the touch? The surface of the ellipsoid of main axes $a = b$ (shorter) and c (longer) is for small prolongation approximately $S = 4\pi a^2(1 + 2c/a)/3$. The water volume is $V_0 = 500\,\mathrm{ml}$, its density is $\varrho = 0.998\,\mathrm{kg{\cdot}l^{-1}}$, and the surface tension is $\sigma = 72.9 \cdot 10^{-3}\,\mathrm{N{\cdot}m^{-1}}$.

Vojta watched ISS on YouTube.

Solution: $T \doteq 105\,\mu\mathrm{J}$

Problem FC ... asteroid field 5 – 3 – 2 – 1

Just imagine an asteroid field where are many asteroids. The asteroids are evenly distributed in numbers according to their radius from zero to some maximum radius $r_{\max}$. What would be the fraction of the mass of the $\delta = 10\,\%$ largest asteroids to the mass of entire asteroid field? Suppose that all asteroids have the same density and there is a very high number of them.

Karel was thinking about asteroids and probability.

Solution: $1 - (1 - \delta)^4 = 34.4\,\%$

Problem FD ... hollow-planet 5 – 3 – 2 – 1

Imagine a hypothetical planet made of a very thin and homogeneous crust. What would be the gravitational acceleration a_g very close above the surface of the planet? Planet has radius R. The surface density of the crust is σ. The gravitational constant is G.

Karel made a lecture about gravitation.

Solution: $a_g = 4\pi\sigma G$

Problem FE ... wire's error 5 – 3 – 2 – 1

We want to determine the electrical resistance of a wire with length $l = 1\,\mathrm{m}$ which can be measured with a measurement error of $\Delta l = 1\,\mathrm{mm}$. The wire has round circular cross-section. The maximum measurement error of wire diameter is $\Delta d = 0.1\,\mathrm{mm}$. What must be the diameter of the wire in order to we would be able to detect relative change $5\,\%$ of the electrical resistance of the wire? *Karel remembered one of the first subjects at the university.*

Solution: $d > 2\frac{\Delta d}{\eta - \frac{\Delta l}{l}} \doteq 4.1\,\mathrm{mm}$

Problem FF ... corrected pendulum 5 – 3 – 2 – 1

We have a pendulum of length $l = 1.00\,\mathrm{m}$. Suppose, the thread is weightless. At the end of the rope, there is a ball of radius $r = 1.00\,\mathrm{cm}$. Suppose, the length of the pendulum is from the axis of rotation to the centre of mass of the ball. The density of the ball is $\varrho = 720\,\mathrm{kg{\cdot}m^{-3}}$. What is our error in not considering the finite size of the ball (it is not a point) and the fact that it is in the air with density $\varrho_\mathrm{v} = 1.30\,\mathrm{kg{\cdot}m^{-3}}$? Exactly, we want $\frac{|T_\mathrm{k} - T|}{T_\mathrm{k}}$, where T_k is corrected period and T is not corrected period of the mathematical pendulum of length l. Neglect air drag. The period of a physical pendulum is $T_\mathrm{f} = 2\pi\sqrt{J/D}$, where J moment of inertia of the pendulum and D is the moment. *Karel remembered very happy times spent with physics practicals.*

Solution: $1 - \sqrt{\frac{1}{1-\frac{\varrho_\mathrm{v}}{\varrho}}}\sqrt{\frac{l^2}{\frac{2}{5}r^2 + l^2}} \approx \frac{\varrho_\mathrm{v}}{2\varrho} + \frac{1}{5}\frac{r^2}{l^2} \doteq 0.092\,\%$

Problem FG ... magnetic tourniquet 5 – 3 – 2 – 1

We have two wires (conductors) in one plane. One of the wires has a shape of a rectangular circuit of sides $a = 1\,\mathrm{m}$ and $b = 3\,\mathrm{m}$ long. Along the longer of the sides of the circuit, there is the second wire in distance of $l = 1\,\mathrm{cm}$. The second wire is very long and there is electrical current going through of $I = 1\,\mathrm{kA}$. Calculate magnetic flux Φ through the circuit. Permeability is $\mu = 1.26 \cdot 10^{-6}\,\mathrm{H{\cdot}m^{-1}}$. *Faleš is simply crazy.*

Solution: $2.78\,\mathrm{mWb}$

Problem FH ... nasty Macocha 5 – 3 – 2 – 1

We have an infinite large thin plate with (area) density $\sigma = 1.17 \cdot 10^{10}\,\mathrm{kg{\cdot}m^{-2}}$ with small hole in it. We throw a stone into the hole with velocity $v = 10\,\mathrm{m{\cdot}s^{-1}}$. Does the stone return in finite time? If yes, when? *Xellos played Fish Fillets.*

Solution: Yes, it will return in $T \doteq 4.1\,\mathrm{s}$.

Problem GA ... linear process 5 – 3 – 2 – 1

We have an ideal two-atom gas. There is a process in this gas going in a straight line in T-V diagram between points $(T_0, 0)$ and $(0, V_0)$. Determine the molar heat capacity of this gas of this process at the point $(T_0/4, 3V_0/4)$? *Xellos stole something for us once again.*

Solution: $13R/6 = 2.167\,R = 18.0\,\mathrm{J{\cdot}mol^{-1}{\cdot}K^{-1}}$

Problem GB ... elder carbon 5 – 3 – 2 – 1

We have a $m = 12\,\mathrm{g}$ sample of carbon. It is from a formerly living animal. It has now the activity of $A = 36\,\mathrm{min}^{-1}$. How old is the sample? The experimentally measured relative ratio of carbon $^{12}\mathrm{C}$ and $^{14}\mathrm{C}$ in living organisms is $k = \frac{N_{^{14}\mathrm{C}}}{N_{^{12}\mathrm{C}}} \approx 1.3 \cdot 10^{-12}$. Half-time of isotope $^{14}\mathrm{C}$ is $T = 5{,}730\,\mathrm{yr}$. Carbon $^{12}\mathrm{C}$ is stable. Neglect the decrease of mass of the sample due to radioactive change. *Karel read a textbook of particle physics.*

Solution: $t = \frac{T}{\ln 2} \ln\left(\frac{\ln 2}{T} k\, m \frac{N_{\mathrm{A}}}{M_{^{12}\mathrm{C}}} \frac{1}{A}\right) \doteq 1.33 \cdot 10^4$ years

FYKOS
Charles University, Faculty of Mathematics and Physics
Institute of Theoretical Physics
V Holešovičkách 2
180 00 Praha 8

www: `http://fykos.cz`
e-mail: `fykos@fykos.cz`

FYKOS is also on Facebook
http://www.facebook.com/FYKOS

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It is part of Public Relations Office activities and is supported by Institute of Theoretical Physics, FMP CU, his employees and The Union of Czech Mathematicians and Physicists.