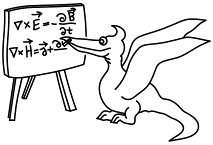

Serial: Mass

If we add the unit of mass – the kilogram to the meter and the second it allows us to use the resulting system of units to describe the motion of bodies and its cause – dynamics. The relationship between the force acting on an object and the change in its motion – acceleration – is precisely defined by Newton's second law, using the mass of the object on which the force acts. Most problems encountered by individuals in physics at the high school level involve dynamics. Let's look at how mass and related quantities are measured.

Mass

The first measurements of mass emerged similarly to the measurement of length with the development of trade in Mesopotamia and Egypt at the end of the third millennium BCE (almost 5,000 years ago). At that time, balanced hanging scales with a set of marked weights were used for weighing. The weights of these scales were whole-number multiples of lighter weights. Units of mass were often derived from common products traded, such as weights of grain or lentils. In addition to the weight itself, the same names were often used to denote the value of goods, which were considered commercial currency. For instance, the British unit "pound" was derived from a unit of weight. Similarly, the carat used for measuring precious stones is derived from the weight of carob seeds. For some products, trade was often conducted based on volume rather than weight alone, such as in the case of liquids, sugar, flour, or other bulk commodities. Therefore, it was natural to convert the measurement of other masses to the volume of the most common liquid – water.

Kilogram was born

The predecessor of the kilogram as we know it today was the French unit of mass called the *grave*. It was defined as the mass of one liter of water in 1793. Later, in 1795, scientists refined the definition of the gram by introducing a reference temperature for ice melting, which allowed them to measure water volume more accurately. This temperature changed with the introduction of the new kilogram in 1799 – measurement at the temperature of maximum water density – and the establishment of the standard *Kilogramme des Archives* made of platinum. The definition was then transferred to the mass of this prototype:

> The kilogram is a unit of mass equal to the mass of the international prototype of the kilogram.

Almost a century later, a new prototype was made from an alloy of 90 % platinum and 10 % iridium to increase material hardness. In 1889, scientists created a new prototype from an alloy of platinum and iridium. They ensured it had an indistinguishable mass from the original reference and transferred the definition to this object. The prototype has a cylindrical shape with a height equal to its diameter to minimize surface area, and it is stored in a safe at the International Bureau of Weights and Measures (BIPM) in Paris under controlled atmospheric conditions. In addition to the prototype, six sister copies are stored in the same vault, and ten working copies are kept in the calibration laboratory at BIPM in Saint-Cloud, France. Besides these, there are nearly a hundred national prototypes, with prototype number 67 in Brno, Czech Republic, and prototypes number 41 and 65 in Slovakia. The mass of these prototypes was determined relative to the original reference prototype, for example, for prototype 67 as $m = 1\,\mathrm{kg} + 0.148\,\mathrm{mg} \pm 0.021\,\mathrm{mg}$.

This standard was subsequently used to calibrate two weights from the austenitic steel, which are used to calibrate the more commonly used precision scales and weights.

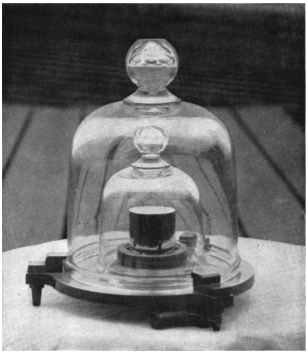

Fig. 1: Prototype of kilogram[1]

Scales

The basic instrument for determining the mass of an object is a scale. As mentioned earlier, the first scales dating back five millennia were simple balance scales. These scales had two pans suspended at the ends of a beam, which was balanced in the center. The mass of an object was determined by comparing it with calibrated weights, a method crucial for the development of trade. In ancient Rome, unequal-arm scales became widespread, allowing the measurement of very small (spices, precious metals, and stones) or very large (building materials) masses, depending on their configuration. In some cases, balancing the scales was achieved by moving the weight or using a combination of multiple approaches.[2] In the 17th century, Roberval scales were invented, consisting of pans placed on a parallelogram-shaped support. Thanks to their design, the measured mass surprisingly does not depend on the position of the object in the pan, allowing for practical weighing without the need for long suspensions—unlike the previous scales, where the suspension of the weighed object ensured that its center of gravity was directly below the suspension point.

So far, we have discussed scales that rely on the equality of the torques of gravitational forces acting on the object and the counterweight. Let m be the mass of the object, M the mass of the counterweight, l_{m} the horizontal distance of the object's center of gravity from the

[1] https://commons.wikimedia.org/wiki/File:Standard_kilogram,_2.jpg

[2] When I was in elementary school, this was how we weighed paper during school collection.

scale axis, and l_M the analogous distance for the counterweight's center of gravity from the scale axis. In equilibrium, the following holds:

$$ml_\mathrm{m}g = Ml_\mathrm{M}g \quad \Rightarrow \quad ml_\mathrm{m} = Ml_\mathrm{M}\,.$$

In the case of isosceles scales, $l_\mathrm{m} = l_\mathrm{M}$, so the counterweight's mass must be the same as the object's mass. For unequal-arm balance, by choosing different lengths for the arms, we can create a conversion factor between the mass of the counterweight and the object suitable for various masses and sets of weights.

Throughout history, other mechanical principles for measuring mass have emerged, based, for example, on the elasticity and deformation of solid materials under the influence of the gravitational force of the weighted object. Analog personal scales are often spring-based— the spring in them shortens or elongates due to the weight, and within the elastic deformation range, Hook's law holds for sufficiently small weights:

$$\Delta l = l\frac{\sigma}{E} = \frac{lmg}{ES} = \frac{mg}{k}\,,$$

where the elongation of the spring (Δl) is proportional to its load. Unlike the previous scales, this does not involve a direct comparison of the masses of two objects. Therefore, we do not need a set of weights, but it is necessary to calibrate its scale.

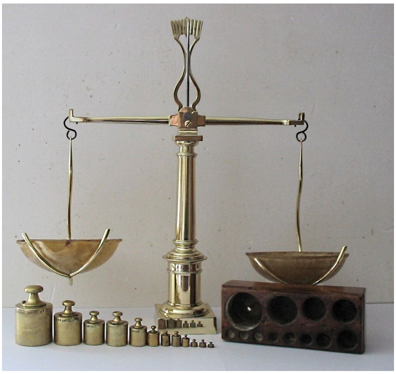

Fig. 2: Balance scales[3]

Fig. 3: Roberval's scales[4]

With the advent of electronics, there has been a significant leap in the precision and capabilities of weighing. Electronic scales convert the measurement of mass using mechanical principles into an electrical signal. When weighing an object, the thin conductive layer experiences elastic deformation, changing its electrical resistance. The internal electronics can then calculate this resistance change into the force applied, and consequently, the mass of the object. An alternative approach involves using a piezoelectric material, where external deformation creates internal voltage, measured by the device. Calibration is crucial for these scales, and they often allow zeroing at a specific load, simplifying readings. Digital scales enable communication with computers and typically have a fast response.

[3] `https://commons.wikimedia.org/wiki/File:Balance_à_tabac_1850.JPG`

[4] `https://commons.wikimedia.org/wiki/File:Balance_Roberval_white.jpg`

One should recognize errors involved in both measuring and interpreting quantities. How we can eliminate or account for these errors is also important. Scales compare the force applied to them with a reference. Incorrect determination of the mass of the weighed object can result from an incorrect reference – whether inaccurate scale calibration (usually dependent on temperature and atmospheric pressure and typically degrades over time) or imprecise determination of the mass of reference weights (e.g., due to dust, dirt, or corrosion). Special tweezers are used to handle weights. The weighted object is also affected by changes in mass – due to reactivity with air, evaporation, or sublimation of volatile substances, or, conversely, condensation of water vapor on cold objects. Inaccuracies in measurements can also result from mechanical components of the scale displacement, an uneven base, excessive friction at suspension points, or sensitivity to vibrations. Additionally, it is crucial to keep in mind that the measured force may not always be equal to mg. The objects being weighed and the weights experience buoyancy in the air, depending on the density of the bodies. Additionally, magnetic fields, electrostatic interactions, or air currents in the vicinity of the scales can affect the objects. The latter can even be caused by air convection induced by the measured object itself if its temperature is significantly different from the surrounding air.

Finally, it is worth noting that scales that do not directly compare the mass of two objects measure the force, which is subsequently converted into mass using the value of gravitational acceleration. This conversion varies depending on the geographic latitude and altitude. As an extreme example, when used on the Moon, equal-arm scales with a set of weights would show correct mass values, while spring or digital scales would indicate only about one-sixth of this value.

Electromagnetism enters the scene

Let us return to the mass prototype, which, as we have just seen, is crucial for calibrating all other weights and masses. During the comparison of prototypes between 1948 and 1989, a gradual deviation in the masses of individual prototypes (from their calibrated masses) was observed, on the order of several tens of micrograms. However, the cause of these changes was not clear, and it was also possible that the entire set of prototypes could have gained/lost a significantly higher mass since comparing the mass of weights with a constant natural mass standard was not possible at the beginning of the 20th century with such precision. Without tying the kilogram to a natural constant, it is not possible to measure mass (in kilograms) with a relative precision much higher than about $1 : 10^{-7}$ – much worse than in the case of length or time. On top of that, this inaccuracy was transferred to the ampere, candela, and mole, and the units derived from them.

Further progress in precision came with the advent of electromagnetism. Scales were initially used to determine the electromagnetic properties of substances using the force of fields. For example, Ampere's scales were used to measure the force between two conductors, which helped determine the value of the electric current flowing through them. On the other hand, the Kibble (Watt) scales work in the opposite way, using electrical quantities to determine weight with extreme precision. The process involves a circular coil of an effective length[5] L that carries an electric current I. Magnetic field lines B run perpendicular to it, so it is subjected to a force $F = BLI$, which is balanced by the weight of the measured object $F = mg$. The

[5] The actual length of the coil can differ. The effective value depends on shapes and orientations in a magnetic field.

need to measure BL – the so-called geometric factor – can be avoided by a second dynamic measurement. The coil is moved in the field with a velocity v, inducing a voltage $U = BLv$ on the coil. By eliminating the BL factor from both equations, we obtain the relationship for the measured mass

$$m = \frac{UI}{gv}\,.$$

During the movement of the coil in a magnetic field, we measure, using a laser interferometer and atomic clocks, the dependence of its position on time, and hence its instantaneous velocity. We measure electric current using an ammeter based on the Josephson effect[6], and voltage using the quantum Hall effect. In the methods of measuring electrical quantities, Planck's constant h comes into play through the von Klitzing constant, the unit of the "quantum" of electrical resistance $R = h/e^2$, and the Josephson constant $J = 2e/h$, the reciprocal of the magnetic flux quantum. We take advantage of the fact that we are not measuring continuous quantities, but rather integral multiples (though high) of certain fundamental values.

We need to measure the gravitational acceleration g in the laboratory with another instrument. An absolute gravimeter with high precision measures the fall of an object in a vacuum tube using atomic clocks and an interferometer, where one arm ends at a retroreflector placed on the falling body. However, to determine the mass of bodies with the highest precision, it is necessary to perform this measurement at multiple locations in the laboratory and build a model of the gravitational field around the measurement location itself.

Today, the Kilogram is defined by the value of Plack's constant as follows

> The kilogram, denoted as kg, is an SI unit of mass defined by a fixed value of Planck's constant $\{h\} = 6.62607015 \cdot 10^{-34}$ expressed in units of $\mathrm{kg{\cdot}m^2{\cdot}s^{-1}}$.

Cosmic masses

We measure the masses of bodies in the universe by their gravitational effects. From Newton's law of gravitation, one of the most widely used relationships – Kepler's third law in an extended form

$$\frac{a^3}{P^2} = M + m$$

for a system where a body of mass m orbits a body of mass M in an elliptical orbit with a semi-major axis of size a and an orbital period P. However, you have surely noticed the missing constants $G/(4\pi^2)$ on the right side of the equation. In this form, the equation is applicable when substituting the period in years, distance in astronomical units, and mass in solar masses. This allows us to easily determine the mass of, for example, Jupiter in units of solar mass using the Galilean moons. As mentioned in the previous section, determining distances in the solar system in Earth units was challenging. However, in the case of mass, we knew the masses of bodies in the solar system in mutual comparison quite accurately. The problem was the knowledge of the value of the gravitational constant. The initial values were derived

[6] a quantum effect occurring at the interface between two superconductors

[7] https://commons.wikimedia.org/wiki/File:Absolute_gravity_measurement_apparatus,_developed_in_1976_at_the_Earthquake_Research_Insitute,_Japan_-_National_Museum_of_Nature_and_Science,_Tokyo_-_DSC07819.JPGhttps://commons.wikimedia.org/wiki/File:Standard_kilogram,_2.jpg

[8] https://commons.wikimedia.org/wiki/File:Nist-4.jpg

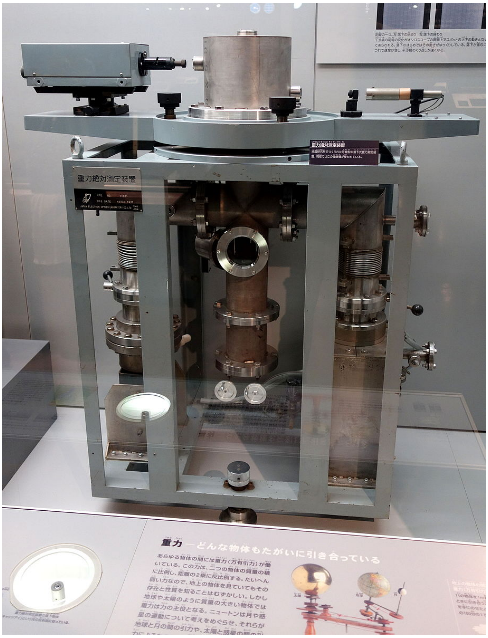

Fig. 4: Absolute Gravity Measurement Apparatus[7]

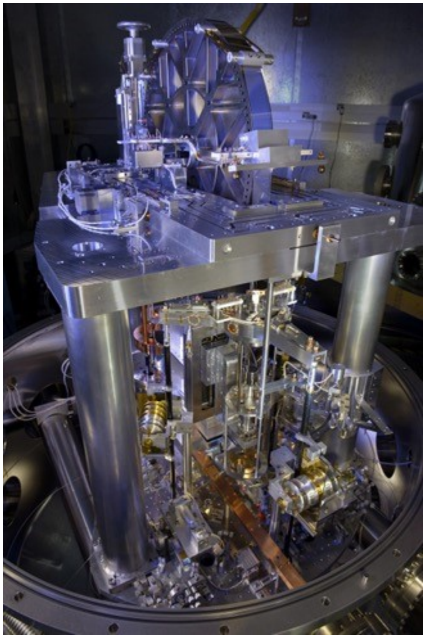

Fig. 5: Kibble Balance[8]

from estimating the mass of the Earth by guessing its average density – essentially an order-of-magnitude estimate. Progress was made in the late 18th century when C. Hutton attempted to determine the mass of Schiehallion mountain in Scotland by measuring the deviation of the vertical direction caused by its gravitational effect. However, it was still necessary to determine the volume of the mountain and its average density to obtain its mass by another method. The deviation from today's value was about 20 %.

The first direct measurement of gravitational interaction between bodies in the laboratory was conducted by Henry Cavendish in 1798. He measured the mutual attractive force between a pair of masses $m = 0.7\,\mathrm{kg}$ suspended at the end of a 1.8 m long arm hanging from a torsion spring and fixed masses of $M = 160\,\mathrm{kg}$. The presence of the heavy weights caused the arm to deflect by approximately four millimeters. To convert the deflection to force, it was necessary to determine the stiffness of the suspension by measuring the torsional oscillations over a period of up to a quarter of an hou – making it essential to isolate the apparatus from external influences such as air currents. The result of the experiment was the determination of the mass of the Earth with an error of about one percent.

Even in the 1950s, the precision of determining the gravitational constant was only about one part per thousand. In astrophysics, masses are commonly measured using solar masses, Jupiter masses, or Earth masses as the unit of measurement. Today, the most accurate measurements of $G = 6.67430(15) \cdot 10^{11}\,\mathrm{m^3{\cdot}kg^1{\cdot}s^2}$ have an uncertainty of about 20 parts per million, while the determination of the product of the solar mass and the gravitational constant is known with a relative precision of $6 \cdot 10^{-11}$ – six orders of magnitude higherFor completeness, let's also mention the value of the Earth's mass $M_\mathrm{E} \doteq 5.97 \cdot 10^{24}\,\mathrm{kg}$ – a fundamental source of practical problems in measurement due to its incomparable value with "human" masses. The masses of stars range in the order 10^{28} to 10^{32} kilograms, and the masses of galaxies reach 10^{44} kilograms.

Atomic masses

Chemists have been interested in the masses of atoms, molecules, and subatomic particles since the time of Lavoisier in the 18th century. However, determining atomic masses directly was difficult, so a new quantity called "the amount of substance" was introduced, with the unit "mole", which we will discuss in a later part. To convert to the mass of a single atom, we need to know Avogadro's constant. Czech scientist Josef Loschmidt provided the first estimate of this constant. He used the relationship between the mean free path of a gas particle and the change in the volume of the gas upon liquefaction. Later, in 1926, Jean Baptiste Perrin won a Nobel Prize for precise measurements of Avogadro's constant. Many methods of that time involved measuring the change in the mass of an electrode during electrolysis, and since 1910, scientists have been using the known value of the charge of the electron. A modern method to determine the atomic mass constant is to determine the number of atoms in a pure macroscopic monocrystal by determining their mutual distances using X-ray diffraction. However, to get the most accurate value of the atomic mass, scientists use natural constants and the mass of the electron measured on the mass spectrometer.

A mass spectrometer is an instrument that measures the ratio of charge to mass of ions. For a charged particle moving at a velocity $\mathbf{v}$ in an electric field with electric intensity $\mathbf{E}$ and a magnetic field with magnetic induction $\mathbf{B}$, the acceleration is given by the equation:

$$\mathbf{a} = \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) ,$$

where m is the mass of the ion and q its charge. Thus, by passing through such a field, we can sort the particles by m/q (more commonly used is m/z, where $z = q/e$ is charge in units of elementary charge) and then detect them on the phosphor screen or digital chip. The most illustrative is the sector mass spectrometer, which uses the dependence of the radius of trajectory R of a particle on the magnetic field strength. In a homogeneous magnetic field with magnitude of B, a particle flying perpendicular to the field lines with a velocity v describes a trajectory in the shape of a circular arc with a radius

$$R = \frac{mv}{qB}\,.$$

Other mass spectrometers work on the principle of measuring the time of flight of a particle accelerated in a purely electric field. An integral part of the instrument is the ionization of the particles themselves.

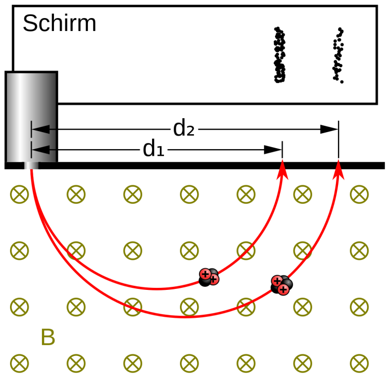

Fig. 6: Mass Spectrometer[9]

In the case of subatomic particles detected in accelerators, their mass m is determined by measuring the total energy E using a calorimeter and the momentum p using a trajectory detector measuring the curvature of the particle's trajectory in the magnetic field and using the well-known special relativity relationship

$$E^2 = p^2c^2 + m^2c^4\,.$$

For subatomic particles, masses are usually expressed in units of $\mathrm{eV{\cdot}c^{-2}}$, where electronvolt eV is the energy imparted to a particle with a charge of one elementary charge by a potential difference of one volt. When accelerating with electric potential, this is a natural unit of energy in such experiments. The lightest material particles with measured masses are electrons, with a mass of $m_\mathrm{e} \doteq 9.11 \cdot 10^{-31}\,\mathrm{kg}$. Regarding neutrinos, at least two types have a nonzero mass. In the case of the electron neutrino, we have an upper limit on its mass, which is $1\,\mathrm{eV{\cdot}c^{-2}} \doteq 1.8 \cdot 10^{-36}\,\mathrm{kg}$.

[9] `https://commons.wikimedia.org/wiki/File:Mass-spectrograph.svg`

Derived units

The addition of units of mass allows the definition of SI units of other mechanical quantities. Force – which is essentially measured by every balance scale – has a unit of newton $1\,\mathrm{N} = 1\,\mathrm{kg{\cdot}m{\cdot}s^{-2}}$. Special names are also given to

- unit of pressure and mechanical stress pascal $1\,\mathrm{Pa} = 1\,\mathrm{N{\cdot}m^{-2}}$, pressure is measured with a manometer by measuring the deformation of the strain member or the height of the liquid level corresponding to a given hydrostatic pressure; for measuring low gas pressures, we must use special methods based on the concentration of particles in the volume;
- energy, work and heat have the unit joule $1\,\mathrm{J} = 1\,\mathrm{N{\cdot}m}$, it is a complex quantity whose magnitude is usually calculated from other measured quantities; the law of conservation of energy is one of the fundamental laws of physics;
- unit of power watt $1\,\mathrm{W} = 1\,\mathrm{J{\cdot}s^{-1}}$, which is also usually determined by calculation, with the exception being electrical power, which we can directly measure with a wattmeter.

Of course, there are many other quantities whose units are simply multiples and ratios of basic or derived units. The measurement of these units depends on the physical phenomena in which they appear. For example, to measure the density of liquids, we need to measure the immersion depth of a densitometer, and we can determine the viscosity of a liquid by measuring the flow rate from a Mariott bottle. Let us also mention the difference between the joule as a unit of energy and the unit of torque, $1\,\mathrm{N{\cdot}m}$ (newton-meter), which are theoretically equal, but the former describes a scalar quantity and the latter a vector quantity.

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