Problem II.1 ... water channel

3 points; (chybí statistiky)

Water flows through a water channel of rectangular cross-section, and width $d = 10\,\mathrm{cm}$. A leaf falls on its surface and starts moving with a velocity of $60\,\mathrm{cm\cdot s^{-1}}$. The height of the water in the channel is $h = 1.3\,\mathrm{cm}$. Estimate how long it will take to fill up a $50\,\mathrm{l}$ bucket. Comment on the assumptions used in comparison with the real situation. *Dodo was cooling his horsefly bite.*

Let us first consider water a perfect fluid. In this case, it would flow throughout the whole volume of the water channel with the same speed v_0 defined in the problem statement. We will base our solution on the relation for the volumetric flow rate

$$Q_V = Sv = \frac{V}{t}\,.$$

After modification, we get an expression

$$t = \frac{V}{dhv_0}\,, \tag{1}$$

from which we get $t = 64\,\mathrm{s}$ after inserting the values given in the problem statement. Thus, filling up a 50-liter bucket would take us $64\,\mathrm{s}$.

If we do not want to consider water a perfect fluid, but on the contrary a fluid with internal friction, viscosity, etc.; then we must think about what model do we want to use to simplify the movement of a real fluid.

In real fluid flow, the molecules closest to the wall of the channel experience the greatest frictional force, and these molecules adhere to the wall, so to speak, forming a non-moving boundary layer. Each successive layer then moves faster. Water moves fastest at its surface on which the leaf, whose speed we measured, fell.

Let us consider a model where the speed of the molecules changes linearly with their height from the bottom of the channel. In this case, we need to calculate the mean speed of the molecules, with which, if we replace the speed of all the layers moving at different speeds, the volumetric flow rate will not change. We calculate this mean speed v_s as an average of the speed of the boundary layer ($v_\mathrm{m} = 0\,\mathrm{cm\cdot s^{-1}}$) and the surface layer

$$v_\mathrm{s} = \frac{v_\mathrm{m} + v_0}{2} = \frac{v_0}{2}\,.$$

When we insert this mean speed into the equation (1) instead of the original speed, we get $t_1 = 2t = 128\,\mathrm{s}$. Hence, according to this model, we would fill up the bucket in 128 seconds. We can see that choosing the right model matters a lot.

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