Problem II.3 ... crane on the raft

6 points; (chybí statistiky)

There is a raft in the middle of the river. The mass of the raft is negligible, and it carries a crane on board. The crane moves boxes of building material of mass m from one river bank to another. In one cycle, the crane loads material at one side of the river, rotates to the other river bank, unloads the material there, and rotates back. Calculate the smallest value of angular displacement of the raft from its original position during one cycle. Approximate the crane by a homogenous cylinder of mass M_j and radius r, and a rotating jib in the shape of a slim rod of length kr. Assume that the velocity of the river and the "friction" between the raft and the water are negligible. *Vojta trained as an engineer at YouTube.*

We will solve this problem by using the law of conservation of angular momentum. The angular momentum relative to the axis of rotation is $L = J\omega$, where J represents the moment of inertia and ω is the angular velocity of the rotating body. Since the total angular momentum of the crane on the raft must remain zero, it has to satisfy

$$J_\mathrm{j}\omega_\mathrm{v} - J_\mathrm{r}\omega_\mathrm{r} - m(kr)^2\omega_\mathrm{r} = 0$$

in the first phase of the motion and

$$J_\mathrm{j}\omega_\mathrm{v} - J_\mathrm{r}\omega_\mathrm{r} = 0$$

in the second phase. In the equations above $J_\mathrm{j} = \frac{1}{2}M_\mathrm{j}r^2$ represents the moment of inertia of the vertical cylindrical part of the crane, $J_\mathrm{r} = \frac{1}{3}M_\mathrm{r}(kr)^2$ represents the moment of inertia of the jib with mass M_r, ω_v is the angular velocity of rotation of the raft, and ω_r is the angular velocity of rotating jib. We will multiply both equations by the time required to rotate the jib by $180\,^\circ$, we will substitute for the moments of inertia and simplify everything.

$$\varphi_\mathrm{v1} = 2k^2\varphi_\mathrm{r}\frac{m + \frac{1}{3}M_\mathrm{r}}{M_\mathrm{j}}\,,$$

$$\varphi_\mathrm{v2} = 2k^2\varphi_\mathrm{r}\frac{\frac{1}{3}M_\mathrm{r}}{M_\mathrm{j}}\,,$$

where $\varphi_\mathrm{r} = \pi\,\mathrm{rad}$ is angular displacement of the jib, and φ_v1, φ_v2 are the angles by which the raft rotates in each phase of the motion. We can calculate the smaller angle by which the raft rotates by subtracting these two values – this will occur when the crane rotates in the opposite direction in each phase of the motion. We will get

$$\Delta\varphi_\mathrm{v} = 2k^2\pi\frac{m}{M_\mathrm{j}}\,\mathrm{rad}\,.$$

Note that the mass of the jib does not appear in the final equation. We can intuitively explain this by saying that the deflections caused by its movement cancel each other out when turning back and forth, and only the mass of the load and the base of the crane will affect the result.

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