Problem IV.4 ... shot telescope

7 points; průměr 4,91; řešilo 56 studentů

We have an astronomical (Keplerian) telescope that we want to launch into space. First, however, we will try it on Earth, where we will measure the magnification Z. How does the distance between the lenses have to change for it to have the same magnification in space? Lenses have a refractive index of n. *Karel gets caught up in those astro-thoughts now and then.*

The Kepler telescope is made up of two biconvex lenses, with the secondary principal focus of the first lens and the first principal focus of the second lens merging. Thus, the distance between the lenses is $f_1 + f_2$. If we move the telescope to a different optical medium, the focal lengths of both lenses change. To get a sharp image, the secondary focus of the first lens and the principal focus of the second lens must again merge, which means that the distance of the two lenses must be $f_1' + f_2'$, where f_1' and f_2' are the focal lengths of the first and second lenses in a vacuum. We can express the focal lengths in the air and the vacuum of the first lens as

$$\frac{1}{f_1} = \left(\frac{n}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$
$$\frac{1}{f_1'} = \left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

Dividing the second equation by the first will give us its ratio

$$\frac{f_1'}{f_1} = \frac{\left(\frac{n}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)},$$
$$\frac{f_1'}{f_1} = \frac{n_0\left(n - n_1\right)}{n_1\left(n - n_0\right)}.$$

We can use this ratio to express f_1' using f_1 and similarly f_2' using f_2

$$f_1' = f_1 \cdot \frac{n_0\left(n - n_1\right)}{n_1\left(n - n_0\right)},$$
$$f_2' = f_2 \cdot \frac{n_0\left(n - n_1\right)}{n_1\left(n - n_0\right)}.$$

We can express the magnification of the Keplerian telescope Z in the air as

$$Z = -\frac{f_2}{f_1}.$$

The magnification of the telescope Z' in a vacuum is

$$Z' = -\frac{f_2'}{f_1'} = -\frac{f_2 \cdot \frac{n_0(n-n_1)}{n_1(n-n_0)}}{f_1 \cdot \frac{n_0(n-n_1)}{n_1(n-n_0)}} = -\frac{f_2}{f_1}.$$

Thus, the magnification does not change. We can express how much the relative distance between the lenses changes by the ratio p, which we can calculate as

$$p = \frac{f_1' + f_2'}{f_1 + f_2} = \frac{f_1 \cdot \frac{n_0(n-n_1)}{n_1(n-n_0)} + f_2 \cdot \frac{n_0(n-n_1)}{n_1(n-n_0)}}{f_1 + f_2} = \frac{n_0\left(n - n_1\right)}{n_1\left(n - n_0\right)}.$$

It is evident that the distance between the lenses must decrease.

To give an idea of what it means, consider the value $n_0 = 1$. The refractive index of the glass is $n = 1.5$, and $n_1 = 1.000\,3$ is the refractive index of air. The ratio's value is $p \doteq 0.999\,1$.

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