Problem V.1 ... flageolet under pressure 3 points; průměr 2,16; řešilo 58 studentů

Vojta plays the cello. He lightly places his finger on a string, tuned to the frequency f, at a distance of $1/n$ of its length from the head of the instrument and sounds it, hearing a tone of fundamental frequency f_1. He then presses the string fully against the fingerboard at the same point and sounds it again. This time the instrument produces a tone of fundamental frequency f_2. Determine the ratio of the frequencies f_1/f_2 as a function of the natural number n. *Vojta reminisces about the cello.*

String emits a sound wave with a wavelength equal to twice the length of the string l in the ground state. We can write it as

$$f = \frac{v}{2l}\,,$$

where v is the speed of the wave propagating in the string. By placing a finger on the string, we can filter out all string vibrations with wavelengths greater than $2l/n$. Therefore, we will hear a tone corresponding to the n-th harmonic frequency of the string, which satisfies

$$f_1 = \frac{vn}{2l} = nf\,.$$

When we pluck the string, we effectively reduce its length to $(1-1/n)l$, from which we can see that the frequency of the plucked tone satisfies

$$f_2 = \frac{v}{2l(1-1/n)} = \frac{n}{n-1}f\,.$$

Thus, we can express the wanted ratio as

$$\frac{f_1}{f_2} = n-1\,,$$

where we intuitively concluded that for $n = 1$ the ratio will be 1. If we pluck the string at $1/2$, the tones produced will be the same; at $1/3$ they will differ by an octave, and at $1/4$ we will get a fifth over an octave, also called a duodecimo.

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