Problem V.2 ... conveyor belt

3 points; průměr 1,47; řešilo 59 studentů

Every second, a material of mass μ falls vertically onto a moving horizontal conveyor belt and falls away at its end. A resistive force $F_{\mathrm{odp}} = kv$, which is directly proportional to the belt speed v through the constant k, acts on the belt. At what speed does thevbelt move if

- *a constant driving force F acts on it?*
- *it is driven by a motor of constant output power P?*

Karel hoped it could be solved.

The belt is decelerated not only by the resistive force but also by the falling material, which we are accelerating. That is true because its momentum changes over time. Every second material with mass μ falls off the belt with the speed of conveyor belt v. Thus, the speed of the material with mass μ must also increase from zero to v every second. The force equals the change in momentum over a given time (that is, one second), thus giving us $F_{\mathrm{mat}} = \mu v$. In summary, the net force required to drive the belt is $F = F_{\mathrm{odp}} + F_{\mathrm{mat}}$. After substituting the known values, we will get

$$F = \mu v + kv\,,$$

which, after adjusting, gives us the speed

$$v = \frac{F}{K + \mu}\,.$$

If a constant power P drives the belt over time t, then the belt does the work W. The new total work W is again the sum of the work required to accelerate the material and the work required to overcome the brake force. Over time t, the falling material increases its kinetic energy thanks to the work

$$W_{\mathrm{k}} = \frac{1}{2}\mu v^2 t,$$

and to overcome the braking force

$$W_{\mathrm{odp}} = F_{\mathrm{odp}} s = F_{\mathrm{odp}} vt = kv^2 t\,,$$

where $s = vt$ is the distance the belt travels in time t.

By adding the previous two equations into one and dividing them by the time t, we will get the resulting velocity equal to

$$v = \sqrt{\frac{P}{\frac{\mu}{2} + k}}\,.$$

For the second part of our problem, we might be tempted to express the total power as $P = P_{\mathrm{mat}} + P_{\mathrm{odp}} = (F_{\mathrm{mat}} + F_{\mathrm{odp}})\, v$, but this would lead us to a different result than the one we got from our energy considerations. The equation $P_{mat} = F_{mat} v$ is invalid because the material is accelerated sequentially, and its parts have different speeds.

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