Problem II.1 ... workout

3 points; průměr 2,31; řešilo 123 studentů

When working out, we often come across workout machines that contain pulleys. Consider the machine in the following figure. What force must be applied on the rope if the velocity of the end of the rope at point A is $v = 0.4\,\mathrm{m\cdot s^{-1}}$ and its direction is downwards? Each pulley has a radius $r = 15\,\mathrm{cm}$ and mass $m = 15\,\mathrm{kg}$. A weight of mass $M = 25\,\mathrm{kg}$ hangs over the free pulley.

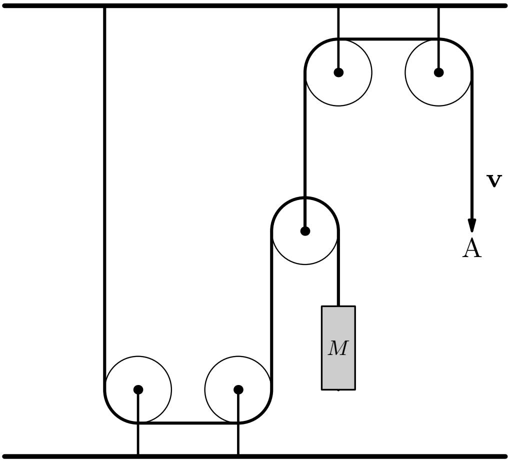

Fig. 1: Scheme of the workout machine.

Dodo went clibming to Smíchoff.

The pulleys in the system have a certain mass, so we have to consider Newton's Second Law of Rotation, where we should consider the torques that will rotate the pulleys. However, according to the problem statement, the rope's end will be pulled by a constant velocity v, which makes the calculation very simple. We have one free pulley that moves uniformly in a straight line – this means that the resulting force on it is zero (according to Newton's Second Law of Motion). Since the pulling rate is constant, so is the angular velocity with which the pulley rotates. We can, therefore, imagine that this free pulley is immaterial and has a weight with mass m suspended from it. There is no need to consider the masses of all other fixed pulleys since they also rotate at a constant angular velocity (all the more so because the two lower pulleys do not rotate at all).

We can express the equality of forces on the free pulley as

$$m \cdot g + T + T = F\,,$$

where F is the force applied at point A, and T is the tensile force of the rope passing over the pulley. Since the pulley does not rotate (as was mentioned above), the tensile force must be equal on both sides of the rope.

The force will be distributed along the length of the rope at a constant rate, as the rope is inelastic. The weight M will also move at a constant speed. The resultant force on the weight M

(according to Newton's Second Law of Motion) will also be zero. Hence, we can calculate the tensile force as

$$T = M \cdot g\,.$$

We substitute this relationship into the first equation to obtain the force

$$F = (m + 2 \cdot M) \cdot g \doteq 640\,\mathrm{N}\,,$$

which must be pulled at point A for the system to move without acceleration.

We could also solve the problem through the Law of Conservation of Energy. The work done by a force F over a length x converts into an increase in the potential energy of the free pulley and the weight. The pulley moves up by x, and the weight M moves up by $2x$.

Nikola Kadlečková
nikola.kadleckova@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.