Problem IV.4 ... a perfect passage?

7 points; průměr 4,05; řešilo 58 studentů

A polarized beam of light coming from a material with refractive index n_1 is incident on a planar interface of a material with refractive index n_2 such that it does not lose intensity after passing through. It then reaches the parallel interface with refractive index n_3, again passing through without any loss, and so on. Find a sequence n_i that satisfies this.

Marek J. met the Brewster's angle

First, we will concentrate on the first part of the problem, in which the polarized beam breaks at the interface of environments n_1 and n_2 so that it does not lose its intensity. This is only possible for p-polarization, when the electric intensity vector is parallel to the plane of incidence, and for a special choice of incident angle – the Brewster angle. The angle is given by a geometrical condition where the *possible* reflected beam must be perpendicular to the refracted beam, as depicted in the figure.

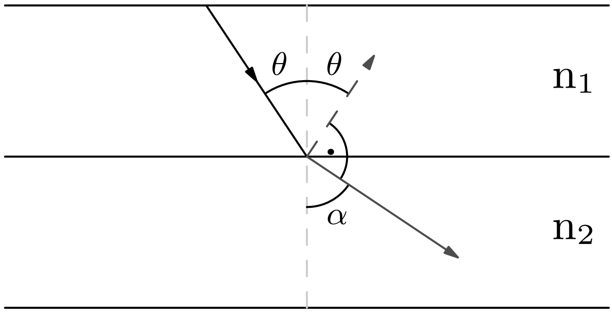

Fig. 1: Nákres situace.

This fact is based on the microscopic conception according to which the electric intensity of the beam oscillates the dipoles in the substrate, and since the electric intensity is perpendicular to the direction of the refracting beam and the dipoles do not radiate in the direction of the oscillation, the entire intensity of the beam passes through the interfaces. The existence of the Brewster angle also follows from the Fresnel equations.[1]

Let us denote the angle of incidence as θ and the angle of refraction as α. From the geometric condition mentioned above, we have $\alpha = \pi/2-\theta$. Snell's law thus gives us the value of Brewster's angle

$$\tan\theta = \frac{n_2}{n_1}\,.$$

Thus, the initial angle of incidence of the ray in the n_1 space is determined, and thus the angle of refraction ($\alpha = \pi/2 - \theta$) is also determined, which is actually the new angle of incidence in the n_2-space. At the interface of n_2 and n_3, however, we again require lossless refraction, and then $\pi/2 - \theta$ is the new Brewster angle for the environments of n_2 and n_3

$$\tan(\pi/2-\theta) = \frac{n_3}{n_2} = \frac{1}{\tan\theta} = \frac{n_1}{n_2}\,,$$

[1] https://en.wikipedia.org/wiki/Fresnel_equations

whence necessarily $n_3 = n_1$. Thus, the looked-for sequence is a repeating n_1 and n_2, which we can choose arbitrarily.

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