Problem I.P ... trains

9 points; (chybí statistiky)

Estimate the consumption of electrical energy for one trip of the IC Opavan train. The train set consists of seven passenger cars, a 151-series locomotive and is capable of reaching a speed of $v_{\max} = 160\,\mathrm{km\cdot h^{-1}}$. For simplicity, consider that all passengers are going from Prague to Opava.

The dwarf takes the train to go home.

Basic Parameters

One of the basic parameters we need to know is the mass of the train. According to the problem statement, the train remains full all the way from the departing station to the final station. Masses of the individual cars and the locomotive are in tab. 1.[1] [2]

Another important parameter that we will need is the drag coefficient C_x, which we will estimate by comparing the shape of the train with the table values. The value we have finally chosen is $C_x = 0.9$. The cross-section of the vehicle S_{TV} will be considered to be rectangle $4\,\mathrm{m} \times 3\,\mathrm{m}$,[3] $S_{\mathrm{TV}} = 12\,\mathrm{m^2}$. The last thing we need to know is the travel time which is 3 hod 43 min according to 2021/2022 time traffic diagram,[4] that is $t = 3.71\,\mathrm{h}$.

Tab. 1: Train composition of the train with masses of empty cars m_{p}, occupied cars m_{o}, car lengths l_{v}, number of passengers n and rotational mass calculated as $m_{\mathrm{r}} = m_{\mathrm{o}}\,(1 + \rho_{\mathrm{r}})$

DV	m_{p} / t	m_{o} / t	l_{v} / m	n / os	m_{r} / t
151	82	82	16.74	-	98.4
Bdpee^{231}	41	47	24.5	72	49.82
Bmz^{226}	48	53	26.4	66	56.18
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Bmz^{226}	48	53	26.4	66	56.18
Bbdgmee^{236}	46	50	26.4	41	53
ARpeer^{61} (ZSSK)	45	47	24.5	24	49.82
Ampz^{146}	47	52	26.4	58	55.12
In total	405	437	197.74	393	474.7

Elevation Profile of the Railway Line

The railway line, like any other linear infrastructure, follows the terrain and adapts to it; for this reason, the line does not run on flat ground but ascends and descends. Train from Prague to Opava begins its journey at the station Praha hl.n. with an elevation of 210 m.a.s.l., then it ascends from the Vltava valley until the Tuklaty station (260 m.a.s.l.), from there it descends to the Elbe valley, and before Kolín it reaches a minimum elevation of just under 200 m.a.s.l. From Kolín the train ascends along the Elbe river and then through the Orlice valley up to the highest

[1] Spolek ŽelPage: Atlas vozů.cz, accessible at `https://www.atlasvozu.cz/`

[2] VAGONY.CZ: tab. 1: values of the coefficient of the rotating masses, available at `https://www.vagony.cz/vagony/energie.html`

[3] Ing. Vilém Hoffmann: Typový výkres rady 151

[4] SŽ s.o.: Jízdní řád, available at `https://www.spravazeleznic.cz/cestujici/jizdni-rad`

point of the journey, Třebovice v Č., with an elevation of 420 m.a.s.l. From there, we descend along the Morava river to the vicinity of the confluence of the Bečva and Morava rivers, i.e. the Dluhonice passing loop (200 m.a.s.l.). Along Bečva we go uphill again just beyond Hranice n. M., namely to the station Bělotín (290 m.a.s.l.), where the inclination of the line changes again, and we descend along the Odra river to the station Ostrava-Svinov (215 m.a.s.l.), where we disconnect from the corridor, and we continue on the single-track railway to the final station Opava (255 m.a.s.l.).[5]

To calculate the energy required for the climb, we will consider a simple model, where we need to lift a body of mass m_o to the height equal to the difference of the individual stations $\Delta h = h_\mathrm{k} - h_\mathrm{z}$, the energy of a single climb will be $E_i = \Delta h_i m_\mathrm{o} g$ viď Tab. 2. The total energy E_prev is calculated as the sum of all the climbs

$$E_\mathrm{prev} = \sum_i \Delta h_i m_\mathrm{o} g\,,$$

$$E_\mathrm{prev} = 1.67\,\mathrm{GJ} = 464\,\mathrm{kWh}\,.$$

Tab. 2: Beginning and end of all the climbs, their elevation h_z and h_k, cumulative elevation gain Δh, and the energy required for individual climbs of the train E

beginning of the climb	h_z / m.a.s.l.	end of the climb	h_k / m.a.s.l.	Δh / m	E / MJ
Praha hl.n.	210	Tuklaty z.	260	50	214
Kolín	200	Třebovice v Č.	420	220	943
Výh. Dluhonice	210	Bělotín z.	290	80	343
Ostrava-Svinov	215	Opava	255	40	171

The Speed Profile of the Line

The train does not travel at the same speed for the entire route. Because of the curves, the state of the infrastructure, and the train control system, it does not reach the maximum speed along the entire route, but it has to slow down and speed up. The speed profile is shown in figure 1.[6]

For these changes in speed, especially for acceleration, energy is needed. When the train travels at speed v_1 and accelerates to speed v_2, it needs energy

$$E = E_{k2} - E_{k1}\,,$$

$$E = \frac{1}{2} m_\mathrm{r} v_2^2 - \frac{1}{2} m_\mathrm{r} v_1^2\,,$$

$$E = \frac{1}{2} m_\mathrm{r} \left(v_2^2 - v_1^2\right)\,,$$

[5] Ing. Pavel Krýže, Ph.D, SŽ s.o.: Nadmořské výšky železničních stanic a zastávek, available at `https://provoz.spravazeleznic.cz/portal/Show.aspx?path=/Data/Mapy/nadm.pdf`

[6] SŽ s.o.: TTP 525A, 501A, 309A, 309E, 305B, 301F

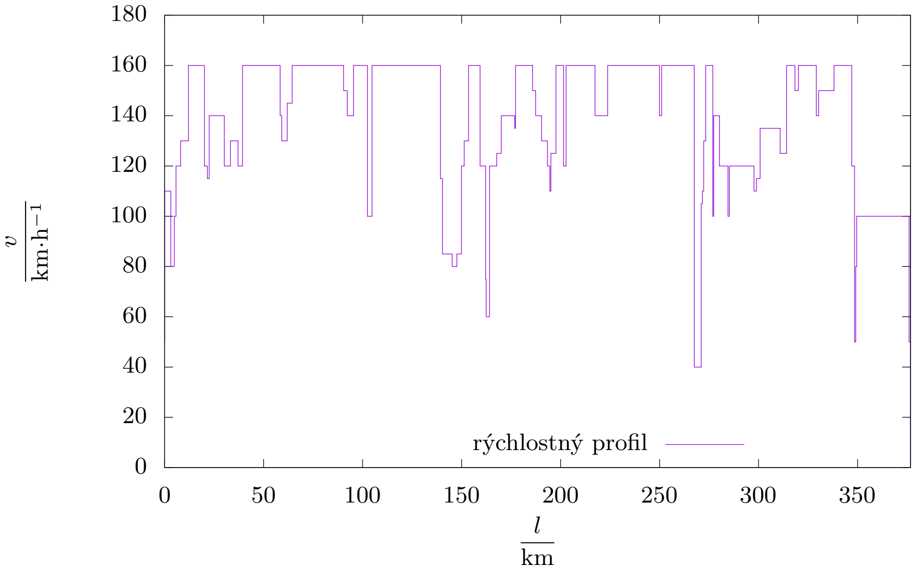

Fig. 1: Graphical representation of the speed profile v of the railway line from the station Praha hl.n., without the train stopping at the stations

which we calculate for the individual jumps in speed where the train accelerates. We are, as always, interested in the total energy

$$E_{\mathrm{vprof}} = \frac{1}{2} m_{\mathrm{r}} \sum_i v_{i2}^2 - v_{i1}^2\,,$$

$$E_{\mathrm{vprof}} = 4.20\,\mathrm{GJ} = 1165\,\mathrm{kWh}\,.$$

This calculation does not include the train stopping at stations, but Opavan stops at stations listed in tab. 3.[7] We proceed as in the previous case with $v_1 = 0$ and $v_2 = v_{\mathrm{zst}}$. In this case we need $E_{\mathrm{zst}} = 816\,\mathrm{MJ} = 227\,\mathrm{kWh}$ for accelerating.

Air Resistance

The train does not travel in a vacuum and is affected by air resistance. We will calculate the air resistance using the speed profile according to formula $F_d = \rho_{\mathrm{vzd}} v^2 C_x S_{\mathrm{TV}}$, where C_x and S_{TV} were determined in the introductory section, the air density $\rho_{\mathrm{vzd}} = 1.29\,\mathrm{km{\cdot}m^{-3}}$ and v is the speed of the train. The energy is calculated as $E_d = F_d l$,[8] where l is the length over which the

[7] SŽ s.o.: TTP 525A, 501A, 309A, 309E, 305B, 301F
[8] We assume $l \gg l_{\mathrm{v}}$.

Tab. 3: Stations of IC Opavan together with speed v_{zst}, at which it accelerates from the station

ŽST	$\frac{v_{\mathrm{zst}}}{\mathrm{km/h}}$
Praha hlavní nádraží	50
Pardubice hlavní nádraží	100
Olomouc hlavní nádraží	140
Ostrava-Svinov	50
Háj ve Slezsku +	100
Opava východ	-

force acts, in our case, the segment with the same speed. The total energy is calculated as

$$E_d = \sum_i \rho_{\mathrm{vzd}} v_i^2 C_x S_{\mathrm{TV}} l_i \,,$$

$$E_d = \rho_{\mathrm{vzd}} C_x S_{\mathrm{TV}} \sum_i v_i^2 l_i \,,$$

$$E_d = 4.12\,\mathrm{GJ} = 1146\,\mathrm{kWh}\,.$$

Vehicle Resistance

Resistance also comes from the wheels themselves, and all moving parts of the bogie, we consider the drag force $F_{\mathrm{v}} = m_{\mathrm{o}} g \left(1,35 + 0,0008v + 0,00033v^2\right)$.[9] The total energy required to overcome this force is then $E_r = F_{\mathrm{v}} l$, where l is the total distance, after inserting the values to the equation we get $E_r = 835\,\mathrm{kWh}$.

Recuperation

Recuperation or return of the reverse current during braking is allowed on the whole line from Prague to Ostrava, but it is forbidden in the Ostrava-Svinov - Opava section. Recuperation is used while braking or traveling downhill. Instead of conventional braking, the train uses “dynamic braking”, where the electric motors act as generators, and the power is returned to the supply line, where it is used by other trains. The SŽDC network does not currently allow current to return to the supply line. At the same time, strict rules apply for recuperation to avoid overloading the network, burning the traction line, or overvoltage in the traction line.

Other “Non-traction” Energy Consumptions: Air Conditioning, Operating Technology, Power Sockets, etc.

Air conditioning in cars consists mostly of two units of electric energy consumption per unit $P \doteq 15\,\mathrm{kW}$.[10] For a total of 7 cars, that makes 14 air conditioning units with total electrical energy consumption of $P_{\mathrm{cl}} \doteq 210\,\mathrm{kW}$, which consumes $E_{\mathrm{cl}} \doteq 666\,\mathrm{kWh}$ per journey.

[9] VAGONY.CZ: tab. 2: rovnice měrného vozidlového odporu `https://www.vagony.cz/vagony/energie.html`
[10] Techklima s.r.o.: skklimy2.crd, available at `http://www.techklima.sk/wp-content/uploads/a4sk.pdf`

During colder parts of the year, it is necessary to heat up the car; for this we consider one unit per car with power $P = 40\,\mathrm{kW}$, which is comparable to the electric energy consumption of the air conditioning.

The car also has lights and other systems; we estimate the total electric energy consumption of these systems $P \doteq 1\,\mathrm{kW}$ per car, in total we get $E_{\mathrm{os}} \doteq 25\,\mathrm{kWh}$ per journey with 7 cars.

Another drain of electric energy are the sockets used by passengers. We estimate the average power to be $P_1 = 15\,\mathrm{W}$, with 393 passengers, this is in total $P_{\mathrm{el}} = 5895\,\mathrm{W}$, which corresponds to an energy consumption of $E_{\mathrm{el}} = 22\,\mathrm{kWh}$.

The locomotive also has auxiliary systems, such as a pneumatic main and auxiliary compressor with an electric consumption per unit of $P \doteq 13\,\mathrm{kW}$. We can estimate the total electric energy consumption of these systems to be $P \doteq 100\,\mathrm{kW}$, which is approximately $E_{\mathrm{hdv}} \doteq 371\,\mathrm{kWh}$.

Total Energy

The power can be divided into "traction", performed by the engines, and "non-traction", auxiliary systems, etc..

The traction powers are, in our case, represented by energies in tab. 4.

The nominal power of a 151-series locomotive is $P_{151} = 4000\,\mathrm{kW}$, the energy calculated by us corresponds to an average power of $P_{\mathrm{t}} = E_{\mathrm{t}}/t$, we get $P_{\mathrm{t}} = 1021\,\mathrm{kW}$, which is much less than the maximum power of the locomotive.

"Non-traction" energies reach values of $E_{\mathrm{nt}} = 1081\,\mathrm{kWh}$, which gives us a total energy required $E_{\mathrm{c}} = E_{\mathrm{nt}} + E_{\mathrm{t}} = 4873\,\mathrm{kWh}$.

Tab. 4: "Traction" energies by type and type of their nominal values

Name	label	$\frac{E}{\mathrm{kWh}}$
Cumulative elevation gain	E_{prev}	464
Changes of speed	E_{vprof}	1165
Stopping at stations	E_{zst}	227
Air resistance	E_d	1146
Vehicle resistance	E_r	835
In total	E_t	3838

Energy Conversion Efficiency

The last thing to consider is energy conversion efficiency. The engines alone have, in today's conditions, efficiency greater than 90 %, the 151-series locomotive is a generation older, and its efficiency can be assumed to be around 80 %.

The load regulation, which is used in the 151-series, generates a large amount of heat and energy loss. The resistors are located on the roof and are cooled by air blown onto them by large fans. The rough ratio of resistor resistance to engine resistance is 1:2; thus, the efficiency is at the order of 66%.

Among other losses can be counted mainly contact resistance on the collector. The estimate of the power transfer efficiency from the traction line to the bogie is approximately 50 %.

Many other factors exist that are not accounted for, for example, service stops, real acceleration/braking, or increased resistance in the curves. At the same time, individual journeys may differ from each other, be it by the driving technique of the driver or also, for example, by the trains in front of/behind this train and thus affecting the voltage in the traction line, etc.

Conclusion

The approximate energy consumed by IC Opavan is 7 MWh.

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports.