Problem II.5 ... magic magnetic stick

10 points; (chybí statistiky)

Consider a thin magnet placed in the middle of a thin hollow rod of length l. The material of the rod is capable of shielding the magnetic field. Just beyond the end of the rod, the magnetic field flux is equal to Φ. Calculate the direction and strength of the magnetic field in a plane perpendicular to the rod passing through its center as a function of the distance r from the rod. *Adam made a blowgun so that he could blow magnets at his classmates in lectures.*

The problem looks very complicated at first glance. In reality, however, we can reformulate the problem in a way that makes the computation fairly easy. First, we realize that the problem is axially symmetric with respect to the axis that is the vertical axis of symmetry of the rod. Moreover, there is no magnetic flux flowing through the walls of the rod (the rod material shields the magnetic field and the rod is very thin). If we look at the situation outside the rod, the magnetic field will behave as if the magnetic flux Φ (or $-\Phi$) is "entering" the space at points on the ends of the rod. The rod and even the magnet now no longer need to be considered. Since the rod is very thin, we can consider the magnetic fluxes $\pm\Phi$ to be homogeneous. We will continue to use $\pm$ to distinguish the ends of the rod.

On a problem so modified, we use very powerful tools called Gauss' theorem and the principle of superposition. For a magnetic field, Gauss' law takes the integral form $0 = \int_S B dS$. Due to the superposition principle, we will calculate the magnetic field for both ends of the rod separately. We take a very small spherical Gaussian area around one end of the rod. The total flux through this area must be zero. The following equation holds

$$\Phi_\pm \mp \Phi = 0,$$

where the opposite sign of the second term is due to the fact that the magnetic flux $\pm\Phi$ enters the Gaussian area.

Since the field for each end of the rod is centripetal, the area through which the magnetic flux $\pm\Phi$ flows is very small and the magnetic flux must be conserved, we obtain the formula

$$|B_1| = |B_2| = B_\pm = \frac{\Phi_\pm}{4\pi R^2} = \frac{\pm\Phi}{4\pi R^2},$$

determining the magnetic field as a function of the radius R of the Gaussian surface. Magnetic induction is perpendicular to this Gaussian surface at every point.

Now it is only the geometry that we have to deal with. Using the figure 1, we express $R = \left(\sqrt{4r^2 + l^2}\right)/2$ and $\cos\alpha = l/\sqrt{4r^2 + l^2}$. The magnetic intensities in the x- and y-axis directions are subtracted, leaving only the z-axis component. Its magnitude will be

$$B = (|B_1| + |B_2|)\cos\alpha = 2B_\pm \cos\alpha = 2 \cdot \frac{\Phi l}{\pi\left(4r^2 + l^2\right)^{3/2}}.$$

The direction of the magnetic field depends on the orientation of the magnet in the rod. When the magnet is oriented as shown in figure 1, the magnetic intensity would point in the $-z$ direction, with the opposite orientation of the magnet it would point in the $+z$ direction. That's all.

A few more notes on the solution:

1. The trick of calculating the whole problem without dealing with the magnet or the rod may seem quite random, on the other hand, the only known quantity characterizing the magnetic field is the flux at the ends of the rod. We know nothing about the magnet whatsoever.

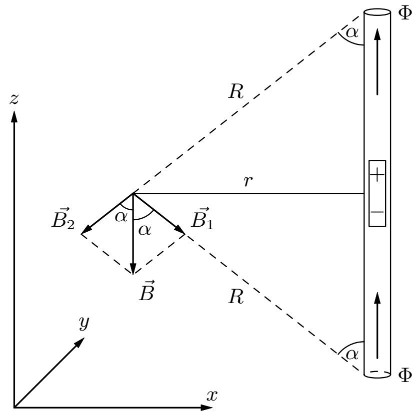

Fig. 1: A sketch of the situation. Here the + sign on the magnet inside the rod represents its north pole, while − represents the south pole.

2. It is not possible for magnetic fluxes $\pm\Phi$ to both enter or exit the rod. The third Maxwell equation would be violated.

3. We can imagine the problem as two magnetic monopoles, put $Q = \Phi\varepsilon$ and solve it using the equations for the electrostatic field. Mathematically, it is equivalent to the solution above, but from the physics point of view, it is incorrect.

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