Problem III.3 . . . randomly you get further

5 points; průměr 4,23; řešilo 86 studentů

In the microworld of cells, there are two types of transport: transport by free diffusion, also known as Brownian motion where the motion uses the energy of the environment directly. The second type, so-called active transport, requires, among other things, a motor protein moving at a constant speed along the cytoskeletal filament. Consider the typical value of the diffusion constant $D \approx 10^{-9}\,\mathrm{cm^2 \cdot s^{-1}}$ and the rate of active transport speed $u \approx 10^{-6}\,\mathrm{m \cdot s^{-1}}$. For which distances is the Brownian motion more time efficient than the active transport? Assume that the transport is happening in only one direction. *Marek J. read Sekimoto.*

The mean distance travelled for a free Brownian particle in 1D is given by

$$\langle x \rangle = \sqrt{2Dt}\,,$$

where t is the time and $\langle \rangle$ denotes the average over different trajectories. A diffusive motion mode is preferred for short times and/or distances since its cause is the surrounding environment. Specifically, the motion is generated by the collisions of the molecules of the environment with the Brownian particle. For longer distances, therefore longer times, randomness can no longer be relied on. The mean velocity $\langle x \rangle / t = \sqrt{2D/t}$ decreases with time. The distance l for which both transport methods are on average equally fast (it takes the same time τ to travel l), is determined as follows

$$\tau = \frac{l^2}{2D} = \frac{l}{u}\,,$$

using the values from the problem statement, we have $l = 2D/u \approx 2 \cdot 10^{-7}\mathrm{m}$. We see that diffusive motion can be preferable to active transport for distances smaller than the order of hundreds of nanometers. Nature indeed takes advantage of this. For example, the distance of roughly $50\,\mathrm{nm}$ between synaptic membranes is overcome by neurotransmitters via diffusion. On the other hand, distances in cells, of the order of micrometres, are typically overcome by molecular motors.

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